55001 Seat No._____

M. Sc. (Part - I) Examination

April / May – 2003

Mathematical Physics, Quantum Mechanics & Computer Programming

Time : **3** Hours] [Total Marks :

Instructions : (1) **All** questions carry **equal** marks.

(2) Symbols have their **usual** meaning.

(3) Calculators can be used.

1 (a) State the Cauchy-Riemann conditions in polar co-ordinates. Use it to find $f(z)$ if $u = r^n \cos n\theta$

(b) If $f(z) = \dfrac{z^4}{(z-1)^4 (z-2)(z-3)}$ then find residue of $z = 1$.

(c) Write two first order and second order irreducible representations of D_3 group. (Symmetry group of an equilateral triangle). If $\Gamma^{(i)}_{(R)}$ and $\Gamma^{(j)}_{(R)}$ are two irreducible representations of D_3 group of order g then verify following mathematical statement of the orthogonality theorem

$$\sum_{R}^{g} * \Gamma^{(i)}_{\alpha\beta}(R)\ \Gamma^{(j)}_{\mu\gamma}(R) = \delta_{ij}\ \delta_{\alpha\mu}\ \delta_{\beta\gamma}\ \frac{g}{d_i}$$

(d) Obtain

(i) $L\left\{\frac{g(t)}{t}\right\} = \int_P^\infty G(u)du$

(ii) $\int_0^\infty t^2 J_0(3t)e^{-2t}dt$

[Hint : $L\{J_0(at)\} = \left(p^2 + d^2\right)^{-1/2}$

$$L\left\{t^n g(t)\right\} = (-1)^n \frac{d^n G(p)}{dp^n}$$

OR

1 (a) Define : Conjugate elements. Obtain conjugate elements of all the elements of D_3 group.

(b) Evaluate : $\int_0^{2\pi} \frac{d\theta}{(a + b\cos\theta)^2}$, $a > b$

(c) Discuss the mapping for

$$w = i + z\, e^{\frac{i\pi}{4}}$$

(d) Discuss Laplace transform of periodic function.

2 (a) Solve by Laplace transform method the following equation

$$t\frac{d^2x}{dt^2}+\frac{dx}{dt}+4tx=0$$

$$x(0)=3 \text{ and } x_1(0)=0$$

(b) Solve the integral equation

$$\varphi(x)=1+x+\int_0^x (x-z)\,\varphi(z)\,dz$$

(c) A covariant tensor has components $xy,\ 2y-z^2,\ xz$ in rectangular co-ordinates. Find its covariant components in spherical co-ordinates.

(d) Discuss stereographic projection and show that

$$\left(\text{residue of } f(Z) \text{ at } Z=\infty\right)$$

$$=-\left(\text{residue of } \frac{1}{Z^2}f\left(\frac{1}{Z}\right) \text{ at } Z=0\right)$$

OR

2 (a) Transform $ds^2=dx^2+dy^2+dz^2$ in spherical polar co-ordinates.

(b) Obtain Green's function for

$$\frac{d^2y}{dx^2}+w^2y=f(x)$$

$$y(0)=0,\quad y(L)=0$$

(c) Obtain the integral form of the following equation

$$y'' + w^2 y = 0$$

$$y(0) = 0, \quad y(b) = 0$$

(d) If $f(a) = \frac{1}{2\pi i} \oint \frac{f(z)}{z-a}\, dz$ then prove

$$f^n(a) = \frac{n!}{2\pi i} \oint \frac{f(z)\ dz}{(z-a)^{n+1}}$$ and use it to find

$\oint_C \frac{\cosh z}{(2\ln 2 - z)^4} dz$. Where C is the circle $|z| = 2$.

[Hint : $L\left(J_0(at)\right) = \left(p^2 + a^2\right)^{-1/2}$

$$L\left(t^n g(t)\right) = (-1)^n \frac{d^n G(\gamma)}{dp^n}$$

$$L\left(y''\right) = p^2 Y - p y_0 - y_0'$$

$$L\left(y'\right) = pY - y_0\ ]$$

3 (a) Discuss the variation theory for ground and excited state. Also discuss the variational theory in which trial wave function is linear combination of known functions.

(b) Apply first order perturbation method for non-degenerate state to obtain ground state energy of He-atom.

OR

3 Taking the example of an appropriate potential function in one dimension, show how, in different regions the wave functions, on the WKB approximation are constructed and connected. Hence obtain the Bohr - Sommerfild quantization condition.

4 Discuss the time dependent first order perturbation theory and obtain the Fermi golden rule.

OR

4 Obtain formal solution of time dependent Schrödinger equation and hence discuss the idea of propagator. Obtain the differential equation for propagator. Also obtain the Lippmann-Schwinger equation.

5 (a) Write down the output of the following program segment

```
void main( )
{   int a=2, b=3, c=4, d=5, f=6, g=66, x, y ;
    printf("%d %d %d %d %c %d \n", a/b, b/a,
                                    c%d, d%c, g, g+1);
    printf("%d %f %e %d \n", – – f, a/5.0,
                                (float)a/d, sizeof(double));
    printf("%d %d %d \n", a+b*c/d, a∗b–c%d, a*(b–c)%d);
    printf("%c\t%c\b%c\n", 'p', 'q', 'r');
    for (x=1, y=7; x !=y; x + +, y– –)
        printf("%d\t", y/x); }
```

(b) Write a program to read a four-digit positive integer number through keyboard, then print sum of first and last digit of the number.

(c) Sum of the squares of first 'n' natural numbers and sum of the cubes of first 'n' natural numbers are given by the following expressions respectively.

$$sum2 = \frac{n(n+1)(2n+1)}{6}$$

$$sum3 = \frac{n^2 (n+1)^2}{4}$$

Write a program to calculate and print sum2 and sum3 for n=5 to 25 in steps of 5. Program also should calculate and print both sums using series expansion for n=10.

OR

5 (a) Distinguish between single character constant, string character constant and escape character.

What do precedence and associativity of operators mean ? Compare the precedence and associativity of the following operators == , <=, &&, || , ++

What are qualifiers (modifiers) ? Mention modifiers for integer type data.

(b) Approximate value for factorial may be obtained by Stirling's formula given below

$$n! = \sqrt{2xn}\ n^n\ e^{-n}$$

Write a program to read n from user and print factorial.

(c) Write a program to accept 50 values through keyboard, find out and print :

(i) highest value

(ii) lowest value

(iii) average of all values

(iv) average of all positive values

(v) average of all negative values.
